

**Exam 1 – Kinematics and Force**

64 PTS

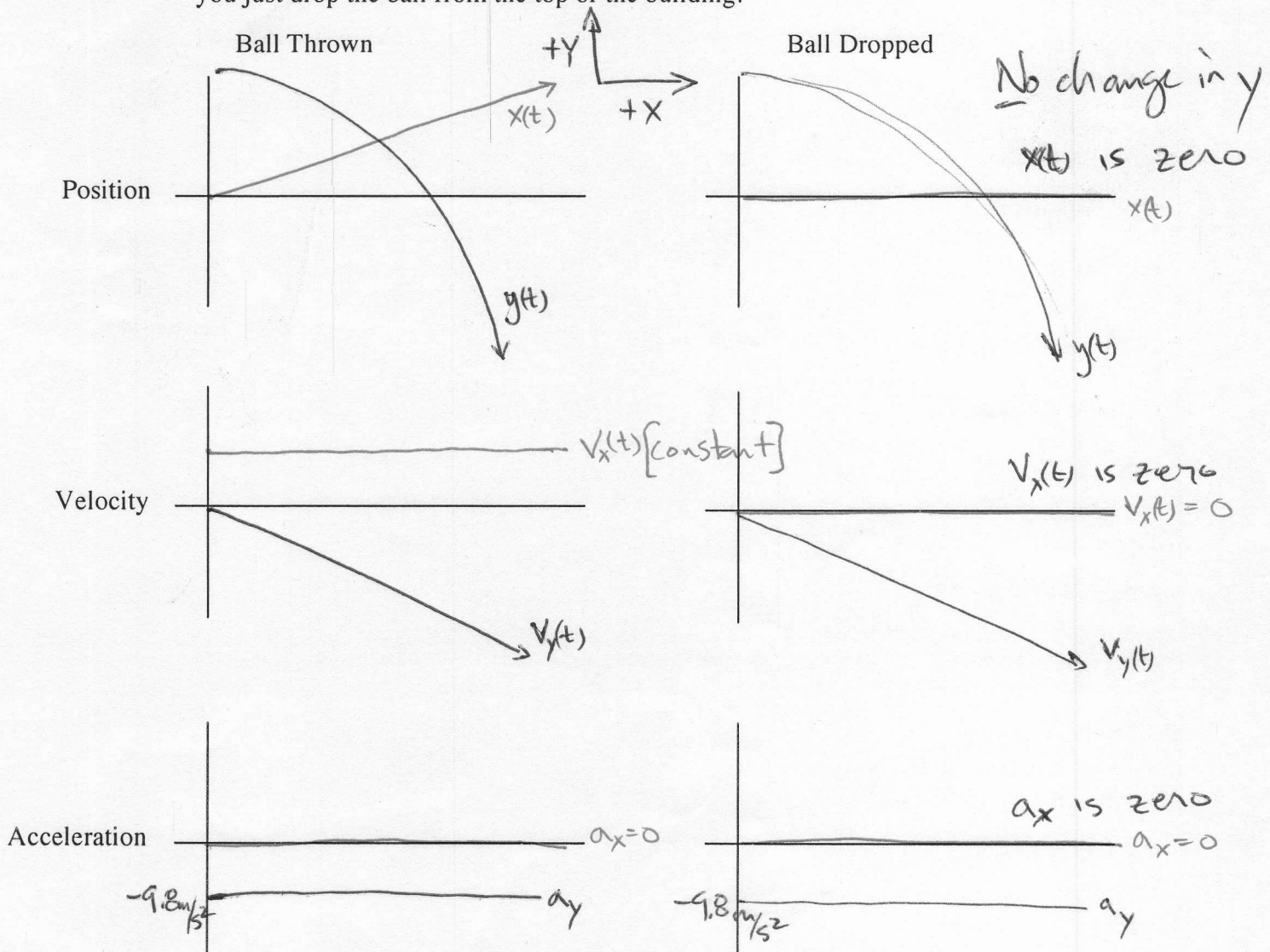
September 24, 2009

This is a closed book examination. There is extra scratch paper available. Your explanation for each problem is worth  $\frac{3}{4}$  of the points. Explain your answers!

A general reminder about problem solving:

1. Draw a picture then create a simplified free body diagram with all forces
2. Write down what you know and pick a coordinate frame
3. Write down what you don't know and/or want to know
4. List mathematical relationships
5. Simplify and solve
6. Check your answer – Is it reasonable? Are units correct?
  - Show all work!

1. [12 PTS] You throw a ball horizontally off the top of a building. Neglecting friction, draw  $x(t)$ ,  $y(t)$ ,  $v_x(t)$ ,  $v_y(t)$ ,  $a_x(t)$  and  $a_y(t)$  for the ball. Indicate what changes on the graphs when you just drop the ball from the top of the building.



2. [4 PTS] You are driving down the freeway and a bug hits your windshield (it is really squashed!). It follows that the force the bug exerts on your car
- is greater in magnitude than the force the car exerts on the bug.
  - is the same magnitude as the force the car exerts on the bug.
  - is smaller in magnitude than the force the car exerts on the bug.
  - can not be compared to the force the car exerts on the bug since you do not know the initial velocity of either object.

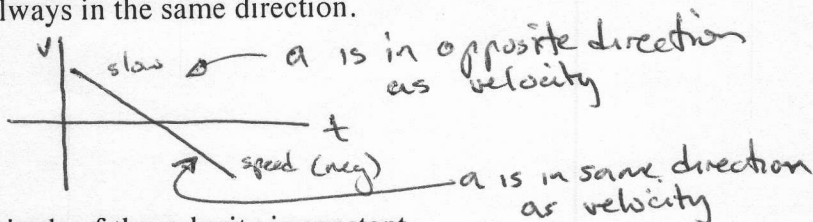
For every force there is an equal and opposite force

Answer the following TRUE or FALSE questions and give an example to demonstrate.

3. [4 PTS] Velocity and acceleration are always in the same direction.

- TRUE
- FALSE

$$\vec{a} = \frac{d\vec{v}(t)}{dt}$$



4. [4 PTS] Acceleration is zero if the magnitude of the velocity is constant.

- TRUE
- FALSE

Circular motion:  $|\vec{v}| = \text{constant}$  but direction changes  
velocity and acceleration are vectors



5. [4 PTS] The net force and acceleration are always in the same direction

- TRUE
- FALSE

$$\sum_i \vec{F} = \vec{F}_{\text{net}} = m\vec{a}$$

6. [4 PTS] When the net force on an object is zero the velocity of the object is also zero.

- TRUE
- FALSE

$$\sum \vec{F} = \vec{F}_{\text{net}} = 0 = m\vec{a} \text{ just means } \frac{d}{dt} v(t) = 0$$

so velocity is constant (does not have to be zero)

7. [4 PTS] Two identical spring-loaded dart guns are fired vertically down at the same time. One fires a regular dart while the other a dart with extra mass. The dart guns exert the same force. Which dart hits the ground first? Ignore air drag.

- The regular dart.
- The dart with extra mass.
- Both darts hit the ground at the same time.

$$\sum_i \vec{F} = F_{\text{gun}} + F_g = ma \quad a = \frac{F_{\text{gun}}}{m} + \frac{mg}{m} = \frac{F_{\text{gun}}}{m} + g$$

$$\text{DART ① } a_1 = \frac{F_{\text{gun}}}{m_1} + g \quad \text{DART ② } a_2 = \frac{F_{\text{gun}}}{m_2} + g$$

$$m_2 > m_1 \text{ so } a_2 < a_1$$

8. [4 PTS] While you pull an object with a constant net force
- the position is constant.
  - the velocity is constant.
  - the acceleration is constant.
  - the acceleration depends on the distance the object is pulled.
  - the acceleration depends on the length of time the force is applied.

$$\sum \vec{F} = \vec{F}_{\text{net}} = m\vec{a} \quad \vec{a} = \frac{\vec{F}_{\text{net}}}{m} \quad \text{so } \vec{a} \text{ is constant if } \vec{F}_{\text{net}} \text{ is constant.}$$

The next two problems can be done on the back of your exam or on additional paper.

9. [12 PTS] A box is stationary halfway up a ramp when you start pushing it ( $F_{\text{push}} = 250 \text{ N}$ ) up the ramp. The ramp is at an angle of  $15^\circ$  and the box has a mass of 50 kg. Ignore friction.
- Draw a force diagram while you are pushing the box.
  - How fast is the box moving when you stop pushing it after 2 meters?
  - How fast is the box moving 5 seconds after you stop pushing it?
  - What is the box's position 5 seconds after you stop pushing it?
10. [12 PTS] You are working as a "reality check" consultant on a movie production. A stunt requires a biker to pedal horizontally off of the roof of one building and land on the roof of a second (shorter) building. The buildings are 40 meters apart and the second building is 30 meters shorter. How fast would the biker need to be moving to make this "jump"? Is this realistic?

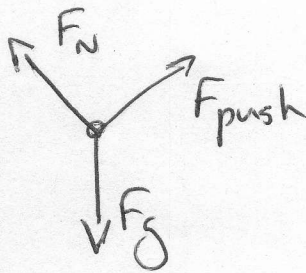
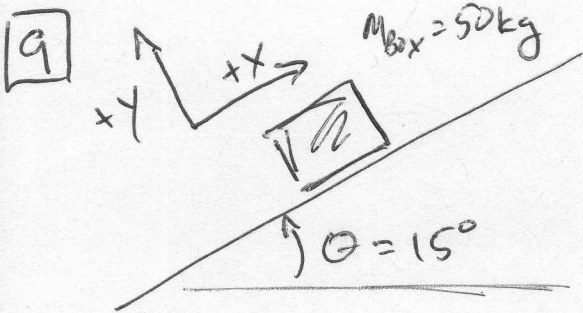
Possibly useful mathematical relationships:

Law of Cosines  $c^2 = a^2 + b^2 - 2ab\cos(\theta)$  which for  $\theta=90^\circ$  is the Pythagorean theorem  $c^2 = a^2 + b^2$   
 Trigonometric identities:  $\sin^2(\theta) + \cos^2(\theta) = 1$ ,  $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$  and  
 $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2\cos^2(\theta) - 1 = 1 - 2\sin^2(\theta)$

Derivative of a polynomial  $\frac{d}{du} Cu^n = nCu^{n-1}$

Anti-derivative (integral) of a polynomial  $\int Cu^n du = \frac{1}{n+1} Cu^{n+1} + \text{const.}$

The Chain Rule  $\frac{d}{dz} f(u) = \frac{d}{dz} u \frac{d}{du} f(u)$



$$\sum F_x = F_{\text{push}} - F_g \sin \theta = m a_x$$

$$\sum F_y = F_N - F_g \cos \theta = 0$$

$$a_x = \frac{F_{\text{push}} - F_g \sin \theta}{m}$$

$$x(t) = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$\frac{d}{dt} x(t) = v_x(t) = v_{x0} + a_x t$$

Combine  $v^2 - v_{x0}^2 = 2 a_x (x - x_0)$

$$x - x_0 = 2m \quad a_x = \frac{250N - 50kg(9.8m/s^2) \sin(15^\circ)}{50kg}$$

$$v_{x0} = 0m/s$$

$$v = \left( 2 \cdot 2m \cdot 2.46 \frac{m}{s^2} \right)^{1/2}$$

$$v_x = 3.14 m/s \text{ at } 2m \text{ up ramp}$$

NOTE: This took  $t = \left[ \frac{2 \Delta x}{a_x} \right]^{1/2}$   
 $\left( \frac{2 \cdot (2m)}{2.46 m/s^2} \right)^{1/2} = 1.28s$

After push  $a_x = -\frac{F_g \sin \theta}{m} = -g \sin \theta$   $F_{\text{push}} = 0N$   
 $= -2.5 m/s^2$

$$v(t=5s) = v_{x0} + a_x t = 3.14 m/s - 9.8 m/s^2 \sin(15^\circ) (5s) =$$

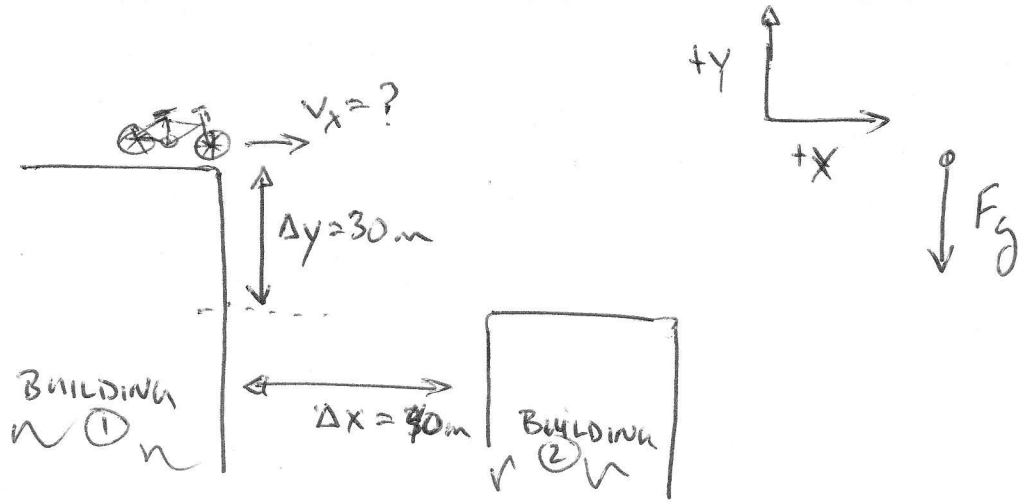
$$v_x = -9.54 m/s$$

$$x(t=5s) = x_0 + v_{x0} t + \frac{1}{2} a_x t^2 = 2m + 3.14 \frac{m}{s} (5s) - \frac{1}{2} (9.8 \frac{m}{s^2}) \sin(15^\circ) (5s)^2$$

$$x = -14m$$

from starting point, so it is further down the ramp

(10)



$$a_x = 0 \quad a_y = -g$$

Determine time to fall 30m. Assume no air drag and totally horizontal jump [~~no~~  $v_y = 0$ ]

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

$$y - y_0 = 30\text{m}$$

$$v_{y0} = 0 \text{ m/s}$$

$$a_y = -g \quad [\text{No Air Drag}]$$

$$t = \left[ \frac{2\Delta y}{a_y} \right]^{1/2} = \left( \frac{2(30\text{m})}{9.8\text{m/s}^2} \right)^{1/2}$$

$$= 2.47\text{s}$$

Determine velocity needed to travel 40m in less than the drop time (2.47s).

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$x - x_0 = 40\text{m}$$

$$v_{x0} = ?$$

$$a_x = 0\text{m/s}^2 \quad [\text{No Air Drag}]$$

$$v_{x0} = \frac{40\text{m}}{2.47\text{s}} \approx 16.2\text{m/s}$$

$$\text{Convert to mph } 16.2\frac{\text{m}}{\text{s}} \cdot \frac{1\text{mike}}{1609\text{m}} \cdot \frac{3600\text{s}}{1\text{hr}} = 36.2\text{mph}$$

This is a very fast speed for a bicycle - but it could be possible if stunt rider got an assist [like from a ramp or rocket]